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The modified London equation, Abrikosov-like vortices and knot solitons in two-gap superconductors

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Abstract

We derive the exact modified London equation for the two-gap superconductor, and compare it with its single-gap counterpart. We show that the vortices in the two-gap superconductor are soft (or continuous) core vortices. In particular, we discuss the topological structure of the finite-energy vortices (Abrikosov-like vortices), and find that they can be viewed as the incarnation of the baby skyrmion stretched in the third direction. Besides, we point out that the knot soliton in the two-gap superconductor is the twisted Abrikosov-like vortex with its two periodic ends connected smoothly. The relation between the magnetic monopoles and the Abrikosov-like vortices is also discussed briefly.

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As an outstanding instance of the condensed matter systems with several coexisting Bose condensates, the two-gap superconductor (TGS) attracts wide interest, both theoretically and experimentally [1]. The discovery of the mapping between a two-flavor Ginzburg–Landau model and a version of the nonlinear $O(3)$ σ-model reveals the topological essence of the TGS [2]. Based on this mapping, many topological solitons in the TGS, including vortices, knot solitons, magnetic monopoles, etc, have been studied [2–5]. In this paper, we derive the exact modified London equation for the TGS, and compare it with its single-gap counterpart. We find that the cores of vortices in the TGS are soft (or continuous). This is distinct from the case in the single-gap superconductor (SGS), where the Abrikosov vortices have hard (or singular) cores. In particular, we discuss the topological structure of the finite-energy vortices (Abrikosov-like vortices), and find that they can be viewed as the incarnation of the baby skyrmion stretched in the third direction. Besides, we point out that the knot soliton in the TGS is the twisted Abrikosov-like vortex with its two periodic ends connected smoothly. The relation between the magnetic monopoles and the Abrikosov-like vortices is also discussed briefly.

We start by reviewing the mapping between a two-flavor GL model and a version of the nonlinear $O(3)$ σ-model. A TGS is described by the two-flavor (denoted by $\alpha = 1, 2$) Ginzburg–Landau free-energy density [2, 3],

$$F = \frac{1}{2m_1} |(\nabla - \mathrm{i}e\mathbf{A})\,\Psi_1|^2 + \frac{1}{2m_2} |(\nabla - \mathrm{i}e\mathbf{A})\,\Psi_2|^2 + \frac{\mathbf{B}^2}{8\pi} + V(|\Psi_{1,2}|^2) + \eta[\Psi_1^*\Psi_2 + \Psi_2^*\Psi_1], \tag{1}$$

where $\Psi_\alpha = |\Psi_\alpha| e^{\mathrm{i}\phi_\alpha}$, $V(|\Psi_{1,2}|^2) = -b_\alpha|\Psi_\alpha|^2 + \frac{c_\alpha}{2}|\Psi_\alpha|^4$ and $\eta > 0$ is a characteristic of the interband Josephson coupling strength. Introduce new variables $\rho^2 = \frac{1}{2}\big(\frac{|\Psi_1|^2}{m_1} + \frac{|\Psi_2|^2}{m_2}\big)$, $\mathbf{C} = \frac{\mathrm{i}}{m_1\rho^2}\big[\Psi_1^*\nabla\Psi_1 - \Psi_1\nabla\Psi_1^*\big] + \frac{\mathrm{i}}{m_2\rho^2}\big[\Psi_2^*\nabla\Psi_2 - \Psi_2\nabla\Psi_2^*\big] + 4e\mathbf{A}$ and $\mathbf{n} = (n_1, n_2, n_3) = (\sin\theta\cos\gamma, \sin\theta\sin\gamma, \cos\theta)$, where $\gamma = (\phi_2 - \phi_1)$ and $\big[\cos\big(\frac{\theta}{2}\big), \sin\big(\frac{\theta}{2}\big)\big] = \big[\frac{|\Psi_1|}{\sqrt{2m_1}\rho}, \frac{|\Psi_2|}{\sqrt{2m_2}\rho}\big]$. Then the original GL free-energy density (1) can be represented as [2, 3]

$$F = \frac{\rho^2}{4}(\nabla\mathbf{n})^2 + (\nabla\rho)^2 + \frac{\rho^2}{16}\mathbf{C}^2 + V(\rho, n_3) + \rho^2 K n_1 + \frac{1}{128\pi e^2}\,(\nabla\times\mathbf{C} + \epsilon_{abc}n_a\nabla n_b\times\nabla n_c)^2\,, \tag{2}$$

where $K \equiv 2\eta\sqrt{m_1m_2}$, $V(\rho, n_3) = A + Bn_3 + Cn_3^2$, and the coefficients A, B, C are given by $A = \rho^2\big[4c_1m_1^2 + 4c_2m_2^2 - b_1m_1 - b_2m_2\big]$, $B = \rho^2\big[8c_2m_2^2 - 8c_1m_1^2 - b_2m_2 + b_1m_1\big]$ and $C = 4\rho^2\big[c_1m_1^2 + c_2m_2^2\big]$. Then the potential term $V(\rho, n_3)$ determines the vacuum value of n_3 to be $\cos\theta_0 \equiv \big[\frac{N_1}{m_1} - \frac{N_2}{m_2}\big]\big[\frac{N_1}{m_1} + \frac{N_2}{m_2}\big]^{-1}$, where $N_\alpha = \langle|\Psi_\alpha|^2\rangle = b_\alpha/c_\alpha$. Furthermore, taking account of the term ρ^2Kn_1, the vacuum value of $\mathbf{n}$ is determined to be $\mathbf{n}_0 = (-\sin\theta_0, 0, \cos\theta_0)$. The models (1) and (2) have four characteristic length scales: condensate coherence lengths ξ_1 and ξ_2, magnetic field penetration length $\lambda = \frac{1}{e\rho}$ and the length scale associated with the interband Josephson effect [3, 4]. For convenience, in the following discussion, we assume that ξ_1 and ξ_2 have the same order of magnitude and define the system coherence length $\xi = \max[\xi_1, \xi_2]$.

According to free-energy density (2), the magnetic field in the TGS is separated into two parts: the contribution from $\mathbf{C}$, which is equal to $\frac{1}{4e}\nabla\times\mathbf{C}$, and the self-induced magnetic field $\tilde{\mathbf{B}} \equiv \frac{1}{4e}\epsilon_{abc}n_a\nabla n_b\times\nabla n_c$, which is originated from the nontrivial electromagnetic interaction between the two condensates [2]. At this point, we might also recall that the magnetic field in the SGS is also separated into two parts: the contribution from the supercurrent, which is the counterpart of the above-mentioned contribution from $\mathbf{C}$, and the magnetic field with a δ-function distribution, which, as we will show below, is the counterpart of the self-induced magnetic field $\tilde{\mathbf{B}}$. From the expression of $\tilde{\mathbf{B}}$, we can see that this part of the magnetic field has a continuous distribution instead of the singular distribution of its counterpart in the SGS. Besides, the expression of $\tilde{\mathbf{B}}$ has an obvious topological meaning, and therefore embodies the topological feature of the system.

For investigating the magnetic-field distribution feature in the TGS, we need the modified London equation for model (1) (or equivalently model (2)). For comparison with that in the SGS, we will first review the derivation of the modified London equation for the SGS which is usually written as [6]

$$\mathbf{B} - \lambda^2\nabla^2\mathbf{B} = \Phi_0\sum_k\int \mathrm{d}\mathbf{x}_k\,\delta^3(\mathbf{x} - \mathbf{x}_k), \tag{3}$$

where λ is the London penetration depth, $\Phi_0 = \frac{2\pi}{e}$ is the standard flux quantum, and the line integral is taken along the kth vortex. During this review process, some corrections to equation (3) will be added. The supercurrent for the SGS is written as $\mathbf{J} = -\frac{ie}{2m}[\Psi^*\nabla\Psi - \Psi\nabla\Psi^*] - \frac{e^2}{m}|\Psi|^2\mathbf{A}$. Combining $\mathbf{B} = \nabla\times\mathbf{A}, \nabla\times\nabla\times\mathbf{B} = \nabla\times(\nabla\cdot\mathbf{B}) - \nabla^2\mathbf{B}$ as well as the Maxwell equations $\nabla\times\mathbf{B} = 4\pi\mathbf{J}$, $\nabla\cdot\mathbf{B} = 0$, we find

$$\mathbf{B} - \frac{m}{4\pi e^2|\Psi|^2}\nabla^2\mathbf{B} + \frac{m}{4\pi e^2}\nabla\frac{1}{|\Psi|^2}\times\nabla\times\mathbf{B} = -\frac{\mathrm{i}}{2e}\nabla\times\left[\frac{\Psi^*}{|\Psi|^2}\nabla\Psi - \frac{\Psi}{|\Psi|^2}\nabla\Psi^*\right]. \tag{4}$$

Using the ϕ-mapping method [7], we can write the right-hand side (RHS) of equation (4) as $-\frac{\mathrm{i}\pi}{e}\delta(\Psi)\nabla\Psi^*\times\nabla\Psi$. Expanding the δ-function $\delta(\Psi)$, we arrive at our modified London equation for the SGS:

$$\mathbf{B} - \frac{m}{4\pi e^2|\Psi|^2}\nabla^2\mathbf{B} + \frac{m}{4\pi e^2}\nabla\frac{1}{|\Psi|^2}\times\nabla\times\mathbf{B} = \Phi_0\sum_k W_k\frac{\mathrm{d}\mathbf{x}_k(s)}{\mathrm{d}s}\int\mathrm{d}s\,\delta^3(\mathbf{x}-\mathbf{x}_k(s)), \tag{5}$$

where W_k is the flux-quantum number of the kth vortex and s is the line parameter. Equation (5) is different than equation (4) mainly in the following aspects: (i) equation (5) can describe the situation including the multi-vortex, (ii) the London penetration depth in equation (5) is a variant and (iii) equation (5) includes an additional correction term, namely the third term in the left-hand side of it. In spite of all these differences, equation (5) retains the main feature of equation (3): excluding the contribution from the supercurrent $\mathbf{J}$, the magnetic field has a δ-function distribution described by the RHS of equation (5).

For the TGS, the supercurrent described in model (1) is written as $\mathbf{J} = -\frac{ie}{2m_1}[\Psi_1^*\nabla\Psi_1 - \Psi_1\nabla\Psi_1^*] - \frac{ie}{2m_2}[\Psi_2^*\nabla\Psi_2 - \Psi_2\nabla\Psi_2^*] - 2e^2\rho^2\mathbf{A}$. As above for the SGS, we can get the following equation for the TGS:

$$\mathbf{B} - \frac{1}{8\pi e^2\rho^2}\nabla^2\mathbf{B} + \frac{1}{8\pi e^2}\nabla\frac{1}{\rho^2}\times\nabla\times\mathbf{B} = -\frac{\mathrm{i}}{4em_1}\nabla\times\left[\frac{\Psi_1^*}{\rho^2}\nabla\Psi_1 - \frac{\Psi_1}{\rho^2}\nabla\Psi_1^*\right] + (1\to 2). \tag{6}$$

Then, using the ϕ-mapping method as above, we can write the RHS of equation (6) as

$$-\frac{\mathrm{i}}{4em_1}\nabla\frac{|\Psi_1|^2}{\rho^2}\times\left[\frac{\Psi_1^*}{|\Psi_1|^2}\nabla\Psi_1 - \frac{\Psi_1}{|\Psi_1|^2}\nabla\Psi_1^*\right] - \frac{\mathrm{i}\pi}{2em_1}\frac{|\Psi_1|^2}{\rho^2}\delta(\Psi_1)\nabla\Psi_1^*\times\nabla\Psi_1 + (1\to 2). \tag{7}$$

Here we note that the term including $\delta(\Psi_1)$ and its Ψ_2 counterpart vanish identically. This shows that the singular part of the magnetic field, which is dominant in the SGS, is replaced by the part of the magnetic field originated from the continuous interaction between the two condensates. Furthermore, from the definition of $\mathbf{n}$, equation (6) can be rewritten in the following compact form:

$$\mathbf{B} - \frac{1}{8\pi e^2\rho^2}\nabla^2\mathbf{B} + \frac{1}{8\pi e^2}\nabla\frac{1}{\rho^2}\times\nabla\times\mathbf{B} = \frac{1}{4e}\epsilon_{abc}n_a\nabla n_b\times\nabla n_c. \tag{8}$$

This expression is the exact modified London equation for the TGS. Comparing equation (8) with equation (5), we find that the self-induced magnetic field $\tilde{\mathbf{B}}$ is indeed the counterpart of the singular magnetic field in the SGS.

Now we turn to the investigation of the topological structure of the finite-energy vortices in model (1) (or equivalently model (2)). In [2], Babaev discussed various vortices in model (1). Among them, only the vortex characterized by $\Delta(\phi_1+\phi_2) \equiv \oint\mathrm{d}\mathbf{l}\cdot\nabla(\phi_1+\phi_2) = 4\pi m$ and $\Delta\gamma \equiv \oint d\mathbf{l}\cdot\nabla\gamma = 0$ (where we integrate over a closed curve around the vortex core) has finite energy per unit length [3, 4]. Such a vortex is an analog of the ordinary Abrikosov vortex in the SGS characterized by $\frac{|\Psi|^2}{m} = \left(\frac{|\Psi_1|^2}{m_1} + \frac{|\Psi_2|^2}{m_2}\right)$, because if both phases $\phi_{1,2}$ change by $2\pi m$ around its core, the vortex will carry m quanta of magnetic flux [3]. In the following, we will refer to such vortices as Abrikosov-like vortices for short. In spite of the similarity between the Abrikosov-like vortices and the ordinary Abrikosov vortices, they have very different topological structures. To see this, let us note the following two points. First, as we have said above, the self-induced magnetic-field distribution in the present system is continuous instead of the singular distribution in the SGS. This implies that the vortices in the present system are soft core vortices while the Abrikosov vortices are the hard core vortices. Second, if we note that an Abrikosov vortex has a vanishing condensate at the center of its core, we may think that the Abrikosov-like vortex has both vanishing condensates at the center of its core. But this is not the case because at the points where both $|\Psi_1|$ and $|\Psi_2|$ vanish, $\mathbf{n}$ can not be well defined. Actually, around such a point, $\mathbf{n}$ has a hedgehog-like distribution, which makes this point corresponding to a magnetic monopole. Such monopoles in the TGS have been noted by Jiang [5]. We will briefly discuss the relation between these monopoles and the Abrikosov-like vortices at the end of this paper.

To be specific, let us consider an Abrikosov-like vortex located along the z-axis. Because the vortex has finite energy per unit length, $\mathbf{n}$ must tend to its vacuum value when the distance away from the center of the vortex core extends to ξ. This boundary condition compactifies the xy-plane into S^2 and makes $\mathbf{n}$ a map: $S^2 \mapsto S^2$. At this point, we can see that the topological stability of the Abrikosov-like vortex originated from $\pi_2(S^2) = Z$ rather than the Abelian topology $\pi_1(S^1) = Z$, which guarantees the topological stability of the ordinary Abrikosov vortex. In 2D, a topological soliton from $\pi_2(S^2) = Z$ is known as a baby skyrmion. So we can say that an Abrikosov-like vortex is an incarnation of a baby skyrmion stretched in the third direction. In general case, we have $N_1 N_2 \neq 0$ or $\cos\theta_0 \neq \pm 1$. Then the curves formed by the zeros of the two condensates are located in the soft core of the vortex, but not at the center which corresponds to $-\mathbf{n}_0$. If $N_2 \to 0$, or $\cos\theta_0 \to 1$, the curves formed by the zeros of Ψ_2 and Ψ_1 will tend to the boundary and center of the soft core, respectively, and the soft core itself will become hard gradually.

With the above topological analysis, we can construct a knot soliton by twisting an Abrikosov-like vortex and connecting its two periodic ends. The topological stability of the knot soliton is guaranteed by the topology $\pi_3(S^2) = Z$. Due to the self-induced feature of $\tilde{\mathbf{B}}$, the Abrikosov-like vortex and the knot soliton can form in the TGS even in type-I limit [2]. According to equation (5), we can make an analysis which concludes that the magnetic field always decays in the magnetic field penetration length of the SGS. As for the TGS, because $\tilde{\mathbf{B}}$ always disperses in the scale of ξ, the similar analysis only applies to the magnetic field originated from $\mathbf{C}$. This leads to the conclusion that because of the interaction of the magnetic field itself, the size of the knot soliton is of order $\xi(\lambda)$ in the type-I(II) limit.

Finally, we comment on the magnetic monopoles mentioned before. From the topological structure of these monopoles, we find that they must be connected by the Abrikosov-like vortices to form the composite solitons. Due to the energy consideration, the monopoles in such a composite soliton are supposed to present in monopole–antimonopole pairs and tend to annihilate. From this analysis, we conjecture that a monopole–antimonopole pair connected by an Abrikosov-like vortex may act as an 'instanton', which could create and annihilate a knot soliton, and therefore tunnel through the barrier between two topologically nonequivalent field configurations. We will leave this subject to future studies.

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